

A Review on suboptimal power allocation schemes for WSN localization

Salar Bybordi, Luca Reggiani

Politecnico di Milano

Dipartimento di Elettronica, Informazione e Bioingegneria

Via Ponzio 34/5, 20133 Milano

Italy

salar.bybordi, luca.reggiani@polimi.it

Abstract: This paper considers a review of two proposed power allocation algorithms for increasing accuracy in localization scenarios, a deeper theoretical analysis and a detailed performance comparison. Appropriate power allocation (PA) among beacons is an effective tool to implement localization with improved precision. At first, a brief review on existing optimal PA strategies is presented. Subsequently, the first PA algorithm is discussed: a function called uncertainty area is defined according to the interaction of beacons in a pair-wise selection procedure. A general selection strategy among allocated transmission powers for each beacon completes the algorithm structure. In the literature, on one hand the commonly made assumption about ranging measures is that their ideal values are equal to their corresponding Cramer-Rao bounds but, on the other hand, at high signal-to-noise ratios, real ranging estimators are characterized by different lower limits on their performance, mainly as a result of maximum sampling rates and computational load available in the sensors. The second PA algorithm develops a type of adaptive PA (APA) directly based on measured SNRs and, consequently, much simpler than other techniques.

Key-Words: Localization, Power Allocation, Ultra-wide Band, Wireless Sensor Networks

1 Introduction

Taking advantage of the development in hardware electronics and communications, Wireless Personal Area Networks (WPANs) and Wireless Sensor Networks (WSNs) have found outstanding importance in recent years. Many interesting applications are foreseen in very diverse areas such as industrial, medical, public services and many other fields. Besides, it is clear that localization of mobile sensors will provide a further strong impulse to new classes of applications. In fact, information about the position of individual nodes, either absolute or in relation to other nodes in the network, is often crucial for a successful fulfillment of the WSN purpose. However, many factors complicate this estimation process, e.g. the absence of a global time reference in asynchronous networks, the presence of non-line-of-sight (NLoS) channels, unknown channel statistics and tight constraints on both energy consumption and node complexity.

Here, the scenario of application is constituted by a set of fixed beacons used for static localization of one target in a limited environment. The possible application areas of this type of algorithm are numerous including, e.g., indoor asset localization using low-complexity amplify-and-forward devices and monitoring systems. Moreover, these technologies

may be used as radio infrastructures for implementing broadband location-based services in environments like railway stations, airports and industrial facilities. The technology considered in the simulations is the Ultra-WideBand Impulse Radio (UWB-IR).

In [1] and [2], a lower bound for target position estimate is derived. Considering that the achieved lower bound, called squared position error bound (SPEB), is a function of the transmit power, a consequent minimization of SPEB with respect to transmit power from each beacon has been pursued in [3–6]. In [3], authors consider the position error in a specific direction, called directional position error bound (DPEB), and they show that SPEB can be seen as a sum of two DPEBs in two orthogonal directions. A consequent minimization of DPEB is achieved along the direction in which the error is maximum (called maximum DPEB or mDPEB). Then the main focus of [4] is on the eigenanalysis of Fisher's Information Matrix (FIM) related to SPEB and both optimal power allocation (PA) and optimal beacon deployment are discussed in order to obtain the minimum SPEB. Taking into account that the parameters involved in the SPEB minimization, like angles and path losses between target and each beacon, are subject to uncertainty in practice, robust versions of the SPEB minimization

are considered in [6]. All the above-mentioned formulations are for active localization where positioning is implemented geometrically by the intersection of different circles in which each beacon is in the center of the corresponding circle. In [7] a similar theoretical analysis using Cramer Rao Bound (CRB) is pursued in order to compute a lower bound for the error of target position estimate and the corresponding power allocation formulation is obtained for passive localization in which localization is implemented geometrically by the intersection of ellipses, each one characterized by a pair of beacons located in their foci.

This paper discusses two proposed localization algorithms in [8,9] with an extensive analysis of both schemes and their mutual comparison. As a matter of fact, appropriate power allocation (PA) among beacons is an effective tool to implement localization with increased accuracy. Before proceeding on the PA algorithms, a brief review on existing, optimal PA strategies in the literature is provided. Then, the first PA is described: a function called uncertainty area is defined w.r.t interaction of beacons in a pair-wise selection procedure and the function is proved to be convex w.r.t. beacons' transmission power. Then a general selection strategy among multiply allocated transmission powers for each beacon completes the algorithm structure. Simulation results are focused on its performance evaluation and its comparison with performance of static localization with optimal PA and without PA (i.e. uniform power allocation or UPA) and the results confirm a promising performance improvement. The results show also that optimal SPEB based PA does not show any advantage w.r.t UPA when the ranging estimator MSE achieves a floor at high SNR. This behavior is evident in practical ranging estimators where increasing transmission powers, leading to a received SNR over a certain SNR threshold, do not provide any additional MSE performance improvement; this effect can be due to numerous causes, including maximum sampling rate and computational load available in the sensors. This is the motivation behind the second investigated PA approach in [9]. Therefore, this PA algorithm is based on distributing transmit power of beacons with SNR above threshold SNR to beacons with SNR below threshold SNR realizing a type of adaptive PA (APA) directly based on measured SNRs and, consequently, much simpler than other techniques. Simulation results confirm that such a simple strategy can be effective at medium low SNR regions, even w.r.t. more sophisticated optimization procedures.

2 A review of PA methods

In this section, a brief review of the proposed PA methods in the literature is presented. Based on CRB for a vector of unbiased estimates, the lower bound for mean square error (MSE) of target position unbiased estimate can be written as [1,5]

$$SPEB = \frac{4 \cdot \sum_j \frac{1}{\sigma_j^2}}{\left(\sum_j \frac{1}{\sigma_j^2} \right)^2 - (C)^2 - (S)^2}, \quad (1)$$

where $C = \sum_j \frac{\cos(2\alpha_j)}{\sigma_j^2}$ and $S = \sum_j \frac{\sin(2\alpha_j)}{\sigma_j^2}$. The

term σ_j^2 is the variance of the distance error in the time of arrival (TOA) based distance estimate from j^{th} beacon w.r.t to the target ($j = 1, \dots, N_B$ with N_B equal to the number of beacons) and α_j is the angle between the target and j^{th} beacon. The column vectors $[x \ y]^T$ and $[x_j \ y_j]^T$ are the positions of the target and j^{th} beacon in 2D coordinates respectively. Since (1) deals with the variance of each of the distance estimates between target and each of beacons, it is worth elaborating the variance of each range measurement using CRB for TOA estimation

$$\sigma_j^2 = E[(\hat{d}_j - d_j)^2] \geq \frac{\chi}{P_{rj}} = \frac{\chi L_j}{P_{tj}}, \quad (2)$$

where $\chi = \frac{c^2 \cdot P_{noise}}{8 \cdot \pi^2 \cdot B^2}$. The terms \hat{d}_j and d_j are the estimated and actual distances between the target and the j^{th} beacon. The variable P_{rj} is the received power from j^{th} beacon measured at the target, B is a measure of the signal bandwidth and P_{noise} is the noise power. Finally the term L_j is the path loss between target and j^{th} beacon, i.e. $P_{rj} = \frac{P_{tj}}{L_j}$. Assuming that variance of distance estimator achieves the CRB, by replacing (2) in (1), we have

$$SPEB = \frac{4\chi \cdot \sum_j \frac{P_{tj}}{L_j}}{\left(\sum_j \frac{P_{tj}}{L_j} \right)^2 - (C')^2 - (S')^2}, \quad (3)$$

where $C' = \sum_j \frac{P_{tj} \cdot \cos(2\alpha_j)}{L_j}$ and $S' = \sum_j \frac{P_{tj} \cdot \sin(2\alpha_j)}{L_j}$.

It is obvious that (3) is a function of two major parameters, i.e. the transmit power $\{P_{tj}\}$ ($j = 1, \dots, N_B$) and signal bandwidth. Consequently, localization MSE can be changed by playing with these two parameters. However, in the sequel signal bandwidth is assumed to be constant. Now, considering (3), the

SPEB turns out to be a convex function of $\{P_{t_j}\}, j = 1, 2, \dots, N_B$ [6]. Consequently, SPEB can be minimized w.r.t. the positive values of major parameters $\{P_{t_j}\}, j = 1, 2, \dots, N_B$ and this minimization problem can be formulated as

$$\begin{aligned} & \underset{\{P_{t_j}\}}{\text{minimize}} \quad \text{SPEB} \\ & \text{subject to} \quad \sum_j P_{t_j} = P_{tot}. \end{aligned} \quad (4)$$

Equ. (4) is the minimization of SPEB w.r.t. a constraint on the fixed total transmitted power (P_{tot}). On the other hand, there can be another formulation based on the minimization of total transmitted power w.r.t. a specific accuracy requirement (ρ):

$$\begin{aligned} & \underset{\{P_{t_j}\}}{\text{minimize}} \quad \sum_j P_{t_j} \\ & \text{subject to} \quad \text{SPEB} = \rho. \end{aligned} \quad (5)$$

The critical concern about these two problems is the fact that other parameters like the angles of target w.r.t. each beacon (α_j) and path losses between target and each beacon are uncertain in practice. In [3], [4] and [5], the two above-mentioned optimization problems are considered under the assumption that there is optimal knowledge of all the parameters; in [6] the authors consider also the case where there is uncertainty in these parameters. All the above-mentioned formulations are for active localization where positioning is implemented by intersection of different circles in which every beacon is in the center of the corresponding circle. In [7], a similar theoretical analysis using CRB is pursued in order to compute a lower bound for the error of target position estimate and the corresponding power allocation formulation using passive localization.

3 System Model and Algorithms

Let $N_B = 3$ fixed UWB transceivers (beacons) with known coordinates (x_i, y_i) be deployed in an indoor environment. The transceivers are equipped with matched filter front ends followed by chip-spaced samplers and the -3 dB system bandwidth is 512 MHz. The localization can be performed at the target, at the beacons or in a central processing station; here we assume that

- the beacons transmit a packet towards the target, which estimates locally the distances from the beacons (with a ranging algorithm), computes locally its position and returns it to the beacons or

to a central processing station (alternatively it returns the distance estimates directly to a central processing station for the whole localization and power allocation computations);

- the algorithm for power allocation is processed at the target or at a central processing station because it needs the data of all the links between the beacons and the target;
- in order to intercept and discuss here the best potential performance of the algorithm, the algorithm is processed with perfect knowledge of the parameters that are needed for deriving the powers to be allocated.

A transceiver pair is formed if two transceivers are within communication range of each other. All the simulations are made at baseband and in discrete time, using complex baseband-equivalent channel models adopted by the IEEE 802.15.4a working group [12]. The used channel models include specific path losses obviously function of the distance and their multipath nature is reproduced by a low-pass filtered tapped delay-line, where signal components arrive at the receiver in independent clusters.

3.1 PA algorithm based on uncertainty area (UCA)

The proposed PA algorithm in [8] is composed by two stages: in the first, it performs PA among beacons in a pair-wise procedure in the sense that it selects two beacons as a pair and accomplishes PA for all available pairs. In our test scenario, characterized by three beacons, each beacon is selected twice in the pair-wise selection procedure (3 pairs). Consequently, there will be two amounts of allocated powers for each beacon. In order to select one of these multiple (in our test scenario two) allocated powers for each beacon, the second part of the algorithm decides between these two allocated powers via a selection procedure described in Section 3.1.2. In the following, an explanation of the two stages of the algorithm is presented.

3.1.1 Power allocation in a pair-wise selection of beacons

The first part of algorithm is based on power allocation among beacons in a pair-wise selection of the beacons. The principle, exploited in the process, is simple. Using Cramer Rao bound (CRB) for TOA estimates, an uncertainty area (UCA) is defined, dependent on the received SNR and on the angle between target and beacon. The UCA for the pair (i, j)

is defined by an expression as

$$A = (\sin(\alpha_i) \cdot \sigma_i + \sin(\alpha_j) \cdot \sigma_j) \times (\cos(\alpha_i) \cdot \sigma_i + \cos(\alpha_j) \cdot \sigma_j), \quad (6)$$

where, according to CRB, σ_i is the standard deviation for a TOA estimator related to i^{th} beacon defined in (2). Considering a fixed total power constraint on the transmit powers, the algorithm minimizes the UCA at each beacons pair. Due to implementation purposes using conventional optimization tools, we present the proof of convexity for the UCA before proceeding with the main core of algorithm.

One way to check whether a multidimensional function is convex or not, is to check Hessian matrix of function from definiteness point of view. If the Hessian matrix is a positive semi-definite matrix, the function is a convex function. Also, if the Hessian matrix is a positive definite matrix, the function is a strictly convex function. To this end, Hessian matrix related to UCA function (A) can be written as

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 A}{\partial p_{t_i}^2} & \frac{\partial^2 A}{\partial p_{t_i} \partial p_{t_j}} \\ \frac{\partial^2 A}{\partial p_{t_j} \partial p_{t_i}} & \frac{\partial^2 A}{\partial p_{t_j}^2} \end{bmatrix}, \quad (7)$$

where

$$\frac{\partial^2 A}{\partial p_{t_i}^2} = \chi \cdot (2acL_i p_{t_i}^{-3} + \frac{3}{4}(ad + bc)\sqrt{L_i L_j} p_{t_i}^{-2.5} p_{t_j}^{-0.5}), \quad (8)$$

$$\begin{aligned} \frac{\partial^2 A}{\partial p_{t_i} \partial p_{t_j}} &= \frac{\partial^2 A}{\partial p_{t_j} \partial p_{t_i}} \\ &= \chi \cdot (\frac{1}{4}(ad + bc)\sqrt{L_i L_j} p_{t_i}^{-1.5} p_{t_j}^{-1.5}), \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial^2 A}{\partial p_{t_j}^2} &= \chi \cdot (2bdL_j p_{t_j}^{-3} \\ &+ \frac{3}{4}(ad + bc)\sqrt{L_i L_j} p_{t_i}^{-0.5} p_{t_j}^{-2.5}), \end{aligned} \quad (10)$$

with $a = \sin(\alpha_i)$, $b = \sin(\alpha_j)$, $c = \cos(\alpha_i)$ and $d = \cos(\alpha_j)$. In order to prove the convexity of A , it should be proved that \mathbf{H} is a positive semidefinite or positive definite matrix. If a matrix is either positive semidefinite matrix or positive definite matrix, all of its eigenvalues should be nonnegative or positive respectively. For the Hessian matrix, we have

$$\det(\mathbf{H}) = v_1 \cdot v_2 > 0 \quad (11)$$

and

$$\text{tr}\{\mathbf{H}\} = v_1 + v_2 > 0, \quad (12)$$

where v_1 and v_2 are the eigenvalues of the Hessian matrix. Equ. (11) and (12) imply that the two eigenvalues are positive. Consequently Hessian matrix \mathbf{H} is a positive definite matrix and it is proved that A is a strictly positive function.

Now, as previously discussed, the proposed UCA is a convex function of its variables (p_{t_i}, p_{t_j}) . Consequently, if UCA has a local minimum, it will be also global minimum. So, the minimization problem for the selected pair (i, j) can be written as

$$\begin{aligned} &\underset{p_{t_i}, p_{t_j}}{\text{minimize}} \quad A \\ &\text{subject to} \quad p_{t_i} + p_{t_j} = P_{tot}. \end{aligned} \quad (13)$$

Since the objective function is a convex function and equality constraint is an affine function of optimization parameters, the formulated problem is a convex optimization problem. Consequently, the minimization problem can be solved by convex optimization solvers like CVX [10].

3.1.2 Selection strategy among multiple allocated powers at each beacon

The second part of algorithm is responsible for an appropriate assignment of one of the allocated powers computed in the first part. Since each beacon is selected by the PA algorithm more than once, after finishing the pairwise selection procedure, the algorithm looks for the best amount of allocated power for each beacon according to the following approach. The algorithm calculates its distances from the beacons using the latest predicted position information. It selects the closest beacon to the target which has the minimum distance. Taking into account that there are two allocated powers for the selected, closest beacon, from two different pair-wise selection procedures, the difference of allocated powers with respect to the value of uniformly allocated power are computed. Assuming beacon i as the selected closest beacon to the target, the power differences ($j = 1, 2$) are calculated according to (in dBm)

$$\Delta P_i^j = P_i^j - P_{uniform}, \quad (14)$$

where $P_i^{1,2}$ are the two allocated powers acquired from two pair-wise selection procedures for beacon i and $P_{uniform}$ is the power for uniform power allocation among beacons. If two computed differences have opposite sign, it means that two different policies should be applied to the transmitted power of

considered beacon. If this condition occurs, the algorithm implements uniform power allocation on all the 3 beacons. If the two computed differences have equal signs, it can be noticed that the allocation, imposed by twice selection of the beacon in the pair-wise selection procedure, is coherent w.r.t. to the increase or decrease of the transmitted power in beacon i . Therefore the algorithm selects the allocated power which has the largest absolute value. The beacon participating in the pair-wise-selection procedure which determines this largest computed difference for the closest beacon, is allocated the power determined via the mentioned pairwise selection. The power at the last beacon, not participating to the pair-wise selection procedure with closest beacon and largest ΔP_i^j , is not changed.

3.1.3 Justification of the general selection strategy via SPEB analysis

Fisher information matrix (FIM) for a static localization scenario can be completely described by its two eigenvalues and the related rotation angle, i.e. $\mathbf{J} = F(\mu_1, \mu_2, \gamma)$ [2]. For a single beacon i , it can be expressed as $\lambda_i \mathbf{J}_r(\alpha_i) = F(\lambda_i, 0, \alpha_i)$ where λ_i is the ranging information intensity (RII) [1]. Obviously, selection of a beacon with the largest RII is the best option [4]. Considering the fact that the considered ranging scheme is TOA based, let us elaborate RII as

$$\lambda_i = \frac{1}{\sigma_i^2}. \quad (15)$$

By substituting (2) in (15), we have

$$\lambda_i = \frac{8 \cdot \pi^2 \cdot B^2 \text{SNR}_i}{c^2}, \quad (16)$$

where SNR_i is the received SNR from i^{th} beacon, which is an inverse function of the distance between beacon and target. Consequently, selection of the closest beacon i.e. with shortest distance will lead to the strongest RII.

3.2 PA algorithm based on threshold SNR

As presented in (4)-(5), optimal PA is achieved by considering ranging measures as necessary inputs of the localization algorithm. In the literature, on one hand the common assumption about ranging measures is that their ideal values are equal to their corresponding Cramer-Rao bounds. In other words, ranging accuracy is proportional to the inverse of SNR and the more SNR increases, the better ranging precision is. On the other hand, at high signal-to-noise ratios,

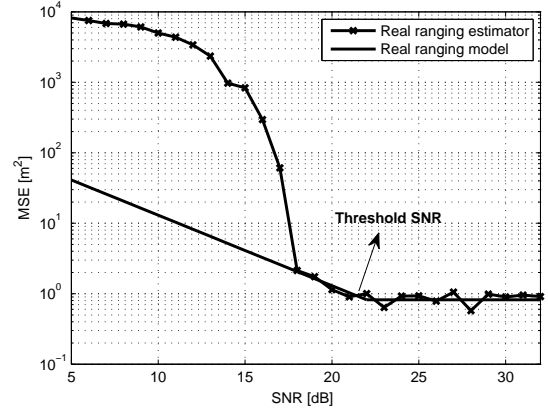


Figure 1: MSE performance of a practical ranging estimator.

real ranging estimators are characterized by different lower limits on their performance mainly as a result of maximum sampling rate and computational load available in the sensors. In this Section, we discuss the impact of real ranging estimators, in which there is a floor in MSE performance, i.e. MSE remains approximately constant while SNR grows over a certain threshold. This behavior is due primarily to maximum sampling rate and computational load achievable in the sensors [9]. Fig. 1 shows MSE performance of soft ranging estimator [13, 14] evaluated at several SNR in a fixed link for residential LoS channel; Therefore, we use a simple model to describe the behavior corresponding to variance of practical ranging required to compute SPEB. The model is defined as

$$\sigma_j^2 = \begin{cases} \frac{c}{\text{SNR}_{lin}^j} & \text{if } \text{SNR}_{lin}^j < \text{SNR}_{lin}^{thr} \\ \frac{c}{\text{SNR}_{lin}^{thr}} & \text{if } \text{SNR}_{lin}^j \geq \text{SNR}_{lin}^{thr} \end{cases} \quad (17)$$

where, considering the particular case of Fig. 1, c has been evaluated equal to 130 according to a simple curve fitting.

It is clear that there are two performance regions: in the first region, estimation accuracy is improved as SNR is increased till a threshold SNR (SNR_{dB}^{thr}) and, in the second region, a floor is observed in the way that estimation accuracy remains almost fixed while SNR exceeds the threshold SNR. Equivalently, in this context increasing transmit power of a beacon with an SNR above SNR_{dB}^{thr} will not provide better accuracy on the corresponding ranging measure and consequently on its contribution to the target localization; so the basic idea considered in this paper is based on distributing transmit power of beacons with SNR above SNR_{dB}^{thr} to beacons with SNR below SNR_{dB}^{thr} realizing a type of adaptive power allocation (APA) based

directly on measured SNRs. The following notations are used in this section: x_{dB} and x_{lin} show variable x in logarithmic and linear scale respectively. For the sake of simplicity, beacons with SNR above SNR_{dB}^{thr} are titled *high* beacons and the beacons with SNR below SNR_{dB}^{thr} are titled *low* beacons. The transmit power of each beacon (in dBm) for uniform power allocation (UPA) is denoted as P_{dBm}^U .

APA Algorithm structure is as follows: first, difference of SNR related to each beacon with respect to the threshold SNR is calculated. Then, based on the sign of each element of the vector δSNR_{dB} , it can be determined which SNR is above or below SNR_{dB}^{thr} . Before proceeding to the main core of algorithm, it is worth elaborating the cases where UPA is assumed as a solution of this algorithm. The first case is the one in which all elements of δSNR_{dB} are positive. In other words, all beacons' SNR are above the SNR_{dB}^{thr} and hence there is no beacon with SNR under which estimation accuracy is improved by increasing the transmit power. The second case occurs when all elements of δSNR_{dB} are negative; as a result, there is no beacon with SNR above SNR_{dB}^{thr} for reducing its transmit power leading to an SNR equal to SNR_{dB}^{thr} . Excluding these cases, the algorithm really works in a scenario with one group of beacons *high* and another *low*. When one group of δSNR_{dB} elements has positive sign while another group contains negative values, the first phase is dedicated to decreasing transmit power of beacons with positive δSNR_{dB}^i in a way that resulting SNR after power cutting is equal to SNR_{dB}^{thr} . After the equalization of transmit power of *high*, the second phase corresponds to distribution of total cut power (P_{cut}^{tot}) over *low* beacons. The priority is with *low* beacon having lowest SNR. Simultaneously with this prior selection, one condition is checked confirming the fact that amount of power required for the SNR of selected *low* beacon to reach SNR_{dB}^{thr} is smaller than total cut power. In fact it is infeasible to distribute an amount of power greater than P_{cut}^{tot} in order to respect the constraint of fixed total transmit power. Each of the *low* beacons, satisfying the aforementioned condition, will be allocated the power so that the related SNR reaches SNR_{dB}^{thr} , keeping in mind that the selection of *low* beacons starts from the one with lowest SNR. After completion of PA for qualified *low* beacons, if there is any remaining P_{cut}^{tot} , it is allocated to the beacon with lowest SNR. The main core of algorithm is iterated to ensure that SNRs related to newly allocated powers does not pass SNR_{dB}^{thr} .

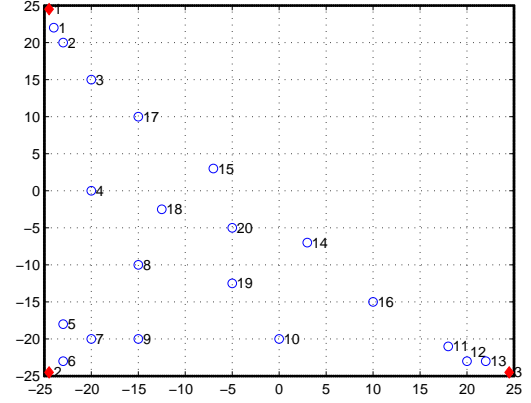


Figure 2: Simulation reference scenario with three beacons. Red dots show the positions of beacons. Blue circles show the point in which the algorithm with PA and without PA is evaluated.

4 Numerical Results

In this test scenario, numerical results are presented for two system categories. In the first category, the ranging estimator MSE is assumed to achieve CRB for TOA estimator (mentioned in (2)) while in the second category, the real ranging model (17) is used as a model for MSE of *soft* ranging. These two categories will be referred as first and second category in the figures and related descriptions respectively. Numerical results are focused on the localization error (i.e. the distance ϵ between estimated and correct locations at each algorithm step). For the sake of simplicity, the localization without PA, localization with PA based on uncertainty area, adaptive PA and localization with optimal PA based on SPEB minimization will be denoted as *WPA*, *UCA*, *APA* and *SPEB based* respectively. Obviously, performance evaluation of *APA* is reported in the plots related to the second category. The simulated scenario is a square room with a side length equal to 50 m in which there are three fixed beacons in the corners (Fig. 2).

In order to show the advantages and the limits of the investigated schemes, simulations are done in two different conditions. Firstly, localization performance of *WPA*, *UCA*, *APA* and *SPEB based* versus increasing transmission power is presented. The second set of results is related to localization performance evaluated in a number of points in the area limited by the beacons. The set of points is chosen in order to understand the different algorithm responses according to the target location; there are points which are close to the beacons and points which are approximately in a symmetric position with respect to the three bea-

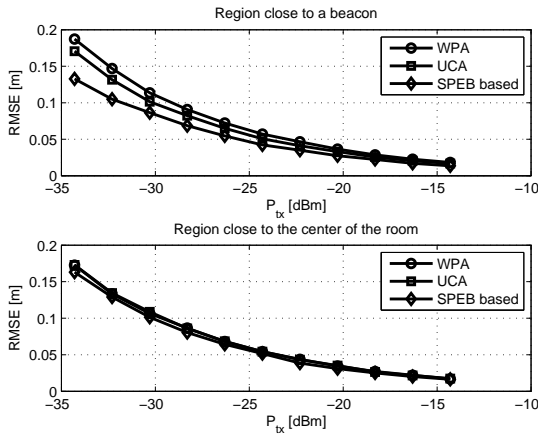


Figure 3: RMSE performance of *WPA*, *UCA* and *SPEB* based power allocation (CRB based TOA estimator).

cons.

The physical parameters of the transmission are taken from the UWB technology. The standard pulse has a reference bandwidth of 512 MHz and the propagation exponent is fixed to a value of $\gamma = 1.79$ according to residential Line-of-Sight UWB channel model. Each receiver noise figure is fixed to 7 dB and each node (beacon or target) respects the UWB transmission spectral power density of -41.3 dBm/MHz.

Fig. 3 represents the RMSE performance comparison among *WPA*, *UCA* and *SPEB* based versus increasing transmit power for the first category evaluated in two regions, one in the proximity of one of the beacons in a circular area with radius equal to 5 m and another one in the neighborhood of the room center in a circular area with radius equal to 5 m keeping in mind that the parts of interest are within the triangle area limited by three beacons. For each value of transmit power (P_{tx}), localization performance is averaged over 300 uniformly distributed points inside the two mentioned regions. This kind of analysis gives us an insight to which areas in the room receive an effective advantage from the application of the considered localization algorithms with PA. In the region close to a beacon, due to a considerable difference in received SNR from each beacon, a considerable performance gap between localization with PA and localization without PA is observed. However, in the region close to the room center, in which all of the received SNR from the beacons have similar values, there is no considerable performance gap between performance of localization with PA (either *SPEB* based or *UCA*) and localization without PA. Focusing on target locations in proximity of beacons where received SNRs have different values, it is ob-

served that the best performance is dedicated to optimal PA of *SPEB* based. Meanwhile *UCA* performs better than the case of uniform power allocation (*WPA*) but not better than *SPEB* based. It is observed that the more the transmission power increases, the smaller the performance gap gets.

Fig. 4 presents performance comparison of *WPA*, *UCA*, *APA* and *SPEB* based for the second category of results, obtained with a real ranging model. The same behavior like the one observed for first category is evident in this plot. It is evident that for small values of transmit power (low SNR regime with SNR_{dB}^{thr} equal to 21 dB) where SNR is in the first performance region of the range estimator, *SPEB* based performs better than *APA*. Also, it is worth mentioning that *APA* shows an advantage w.r.t localization without PA. By increasing transmit power, the performance gap between *SPEB* based and *APA* decreases and the two performance curves intersect at a point P_{int} . This behavior is due to the fact that SNR approaches the threshold SNR and consequently ranging MSE achieves the floor. By increasing P_{tx} over P_{int} , the performance gap changes in favor of *APA*. It is interesting that by increasing further transmit power or, equivalently, moving into the second performance region of the ranging estimator, there is no performance advantage by means of PA. *UCA* shows an almost stable performance advantage over *WPA* for all values of P_{tx} but not better than *SPEB* based. It outperforms *APA* for small values of P_{tx} while *APA* shows better performance for greater amounts of P_{tx} . We can also observe that, in this second category, the optimal *SPEB* approach does not show any advantage over a certain transmission power. Consequently, in the successive figures related to the real ranging model, localization performance related to *SPEB* based is not reported.

The localization RMSE in the first category results, evaluated in the points in Fig. 2 is shown in Fig. 5. The best performance is related to *SPEB* based, which is the optimal PA strategy in all the points. Comparing performance of *WPA* and *UCA*, the considerable performance gap between localization appears in target locations in the vicinity of beacons like 1, 6 and 12. In other points, *UCA* performance is equal to that of *WPA* or slightly better confirming the fact that when the target approaches a beacon, *UCA* achieves performance greater than *WPA*. Otherwise, the PA strategy for *UCA* tends to implement uniform PA leading to performance equivalent to *WPA*. This behavior leads to better localization performance fixed the same total transmit power at beacons or energy savings once fixed the performance level. The same analysis for a practical ranging estimator i.e. *soft* is presented in Fig. 6. All the conclusions made for per-

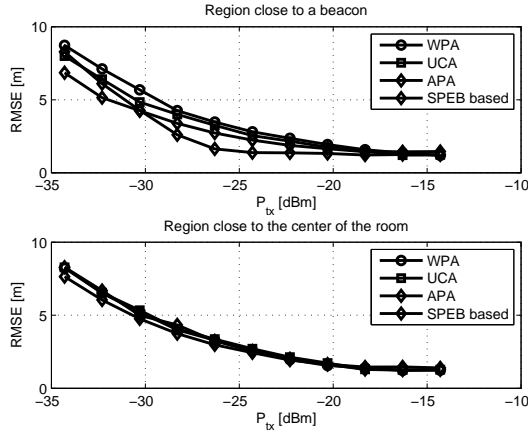


Figure 4: RMSE performance of *WPA*, *UCA*, *APA* and *SPEB* based (real ranging model).

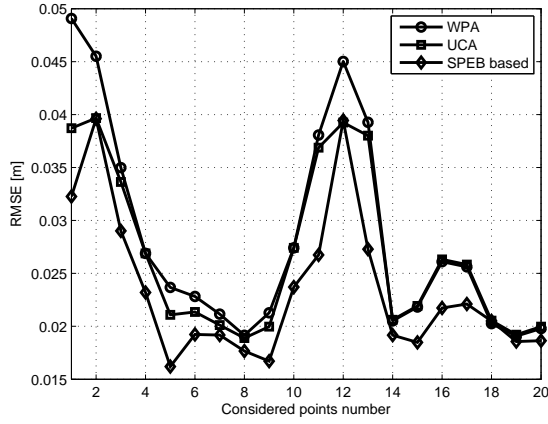


Figure 5: Localization performance of *WPA*, *UCA* and *SPEB* based in assumed points inside the indoor environment delimited by three beacons (CRB based TOA estimator).

formance comparison of *UCA* and *WPA* for Fig. 5 are also valid for Fig. 6. Additionally, it is evident that *APA* outperforms both *UCA* and *WPA* in the points located in vicinity of beacons while it shows the equal performance w.r.t *UCA* and *WPA* in the points located in an almost symmetric geometry w.r.t beacons.

Fig. 7 depicts in more detail some numerical results, reporting the cumulative density functions (CDFs) of the distance error for the *WPA*, *UCA* and *SPEB* based for the first category results. The plots reveal the CDFs of distance error at two points, 1 (Fig. 7.a) and 19 (Fig. 7.b), one close to the beacon located at the top corner of the room and one located almost in the middle of the room. As it is expected, the localization advantage, with *UCA*, is present only at the point closer to the beacon while it is absent in

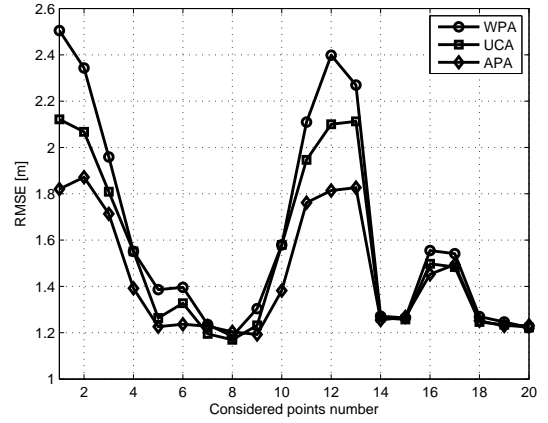


Figure 6: Localization performance of *WPA*, *UCA* and *APA* in the points inside the indoor environment delimited by three beacons and related to the real ranging model.

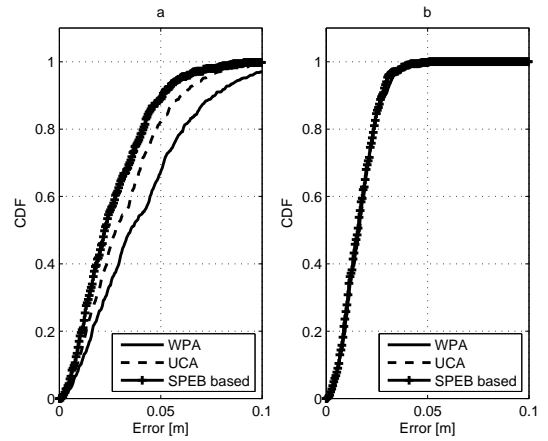


Figure 7: CDF plots of localization error related to CRB based TOA estimator in two points.

the other point, where the best power allocation is just the uniform one (this is obvious also by using simple geometrical considerations). Fig. 8 reports the same analysis for second category where real ranging model is used. The same conclusion on the comparison between *UCA* and *WPA* is also valid for second category. Additionally, *APA* outperforms *UCA* and *WPA* in the point 1 close to vicinity of a beacon while in the point close to room center its performance is equal to that of *UCA* and *WPA*, resulting from the fact that *APA* power allocation policy approaches the uniform power allocation.

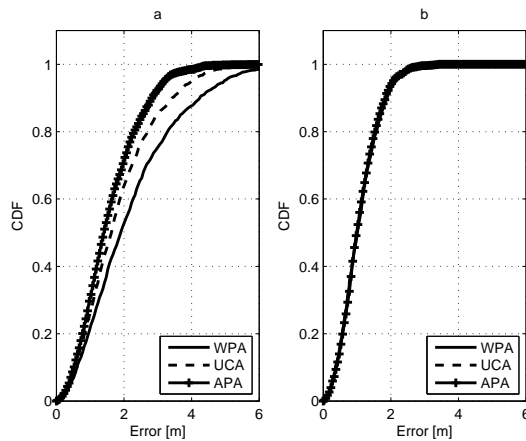


Figure 8: CDF plots of localization error related to second category results in two points.

5 Conclusion

In this paper, we presented a review of two PA algorithms for localization scenarios proposed in [8, 9] with further analytical justification and their mutual performance comparison. The first algorithm aims to minimize a convex function of transmission powers in a pair-wise selection of beacons. Simulation results confirm the fact that optimal PA approach does not show advantage in the case of practical ranging estimator where MSE performance does not improve by increasing transmission power over a certain SNR threshold. This behavior is a result of implementation issues like maximum sampling rate and computational load available in sensors. The second algorithm aims to equalize the received SNR to the certain threshold SNR leading to an adaptive PA (APA).

References:

- [1] Y. Shen and M. Z. Win, "Fundamental limits of wideband localization Part I: A general framework," *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 4956–4980, Oct. 2010.
- [2] Y. Shen, H. Wymeersch and M. Z. Win, "Fundamental limits of wideband localization Part II: Cooperative networks," *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 4981–5000, Oct. 2010.
- [3] W. W.-L. Li, Y. Shen, Y. J. Zhang, and M. Z. Win, "Efficient anchor power allocation for location-aware networks," in *Proc. IEEE Int. Conf. Commun.*, Kyoto, Japan, Jun. 2011, pp. 1–6.
- [4] Y. Shen and M. Z. Win, "Energy efficient location-aware networks," in *Proc. IEEE Int. Conf. Commun.*, Beijing, China, May 2008, pp. 2995–3001.
- [5] Y. Shen, W. Dai and M. Z. Win, "Optimal power allocation for active and passive localization," in *Proc. IEEE Global Telecomm. Conf.*, Anaheim, CA, USA, 2012, pp. 3713–3718.
- [6] W. L. Li, Y. Shen, Y. J. Zhang, and M. Z. Win, "Robust power allocation for energy-efficient location-aware networks," *IEEE/ACM Trans. Netw.*, vol. 21, no. 6, pp. 1918 – 1930, 2013.
- [7] H. Godrich, A. Petropulu and H. V. Poor, "Power allocation strategies for target localization in distributed multiple-radar architectures," in *IEEE Trans. Signal Process.*, vol. 59, no. 7, pp. 3226–3240, Jul. 2011.
- [8] S. Bybordi and L. Reggiani, "Algorithm for power allocation in localization processes," *International Symposium on Wireless Communication Systems (ISWCS)*, Aug. 2013.
- [9] S. Bybordi and L. Reggiani, "Impact of real ranging on algorithms for power allocation in localization processes," *The IEEE International Conference on Ultra-Wideband (ICUWB)*, Sep. 2014, Paris-France.
- [10] M. Grant S. Boyd, "CVX: Matlab software for disciplined convex programming," [Online]. Available: <http://stanford.edu/boyd/cvx>
- [11] L. Reggiani and S. Bybordi, "Performance trade-offs for energy efficient localization based on EKFs," *International Symposium on Wireless Communication Systems (ISWCS)*, pp. 501–505, Aug. 2012.
- [12] A.F. Molisch, K. Balakrishnan, D. Cassioli, C.-C. Chong, S. Emami, A. Fort, J. Karedal, J. Kunisch, H. Schantz, U. Schuster, K. Siwiak, *IEEE 802.15.4a channel model - final report*, IEEE P802.15 WPAN, July 2005.
- [13] M. Rydstrom, L. Reggiani, E. G. Strom, and A. Svensson, "Suboptimal soft range estimators with applications in UWB sensor networks," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 4856–4866, Oct. 2008.
- [14] M. Rydstrom, E. G. Strom, A. Svensson and L. Reggiani, "An Algorithm for Positioning Relays and Point Scatterers in Wireless Systems," *IEEE Signal Processing Letters*, vol. 15, pp. 381–384, 2008.